The Mathematics of Smart Cities

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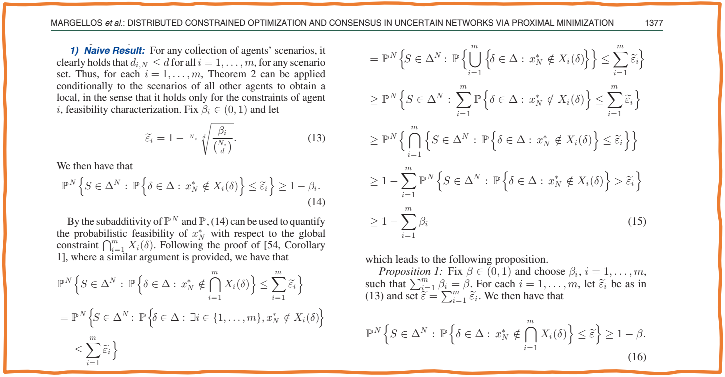
Disclaimer

- System's engineer by education and my research looks like this



MARGELLOS *et al.*: DISTRIBUTED CONSTRAINED OPTIMIZATION AND CONSENSUS IN UNCERTAIN NETWORKS VIA PROXIMAL MINIMIZATION 1377

1) Naive Result: For any collection of agents' scenarios, it clearly holds that $d_{i,N} \leq d$ for all $i = 1, \dots, m$, for any scenario set. Thus, for each $i = 1, \dots, m$, Theorem 2 can be applied conditionally to the scenarios of all other agents to obtain a local, in the sense that it holds only for the constraints of agent i, feasibility characterization. Fix $\beta_i \in (0,1)$ and let

$$\widetilde{\varepsilon}_i = 1 - \sqrt[N_i - d]{\frac{\beta_i}{\binom{N_i}{d}}}. \qquad (13)$$

We then have that

$$\mathbb{P}^N\left\{S \in \Delta^N : \mathbb{P}\left\{\delta \in \Delta : x_N^* \notin X_i(\delta)\right\} \leq \widetilde{\varepsilon}_i\right\} \geq 1 - \beta_i. \qquad (14)$$

By the subadditivity of $\mathbb{P}^N$ and $\mathbb{P}$, (14) can be used to quantify the probabilistic feasibility of x_N^* with respect to the global constraint $\bigcap_{i=1}^m X_i(\delta)$. Following the proof of [54, Corollary 1], where a similar argument is provided, we have that

$$\mathbb{P}^N\left\{S \in \Delta^N : \mathbb{P}\left\{\delta \in \Delta : x_N^* \notin \bigcap_{i=1}^m X_i(\delta)\right\} \leq \sum_{i=1}^m \widetilde{\varepsilon}_i\right\}$$

$$= \mathbb{P}^N\left\{S \in \Delta^N : \mathbb{P}\left\{\delta \in \Delta : \exists i \in \{1, \dots, m\}, x_N^* \notin X_i(\delta)\right\} \leq \sum_{i=1}^m \widetilde{\varepsilon}_i\right\}$$

$$= \mathbb{P}^N\left\{S \in \Delta^N : \mathbb{P}\left\{\bigcup_{i=1}^m \left\{\delta \in \Delta : x_N^* \notin X_i(\delta)\right\}\right\} \leq \sum_{i=1}^m \widetilde{\varepsilon}_i\right\}$$

$$\geq \mathbb{P}^N\left\{S \in \Delta^N : \sum_{i=1}^m \mathbb{P}\left\{\delta \in \Delta : x_N^* \notin X_i(\delta)\right\} \leq \sum_{i=1}^m \widetilde{\varepsilon}_i\right\}$$

$$\geq \mathbb{P}^N\left\{\bigcap_{i=1}^m \left\{S \in \Delta^N : \mathbb{P}\left\{\delta \in \Delta : x_N^* \notin X_i(\delta)\right\} \leq \widetilde{\varepsilon}_i\right\}\right\}$$

$$\geq 1 - \sum_{i=1}^m \mathbb{P}^N\left\{S \in \Delta^N : \mathbb{P}\left\{\delta \in \Delta : x_N^* \notin X_i(\delta)\right\} > \widetilde{\varepsilon}_i\right\}$$

$$\geq 1 - \sum_{i=1}^m \beta_i \qquad (15)$$

which leads to the following proposition.

Proposition 1: Fix $\beta \in (0,1)$ and choose β_i, $i = 1, \dots, m$, such that $\sum_{i=1}^m \beta_i = \beta$. For each $i = 1, \dots, m$, let $\widetilde{\varepsilon}_i$ be as in (13) and set $\widetilde{\varepsilon} = \sum_{i=1}^m \widetilde{\varepsilon}_i$. We then have that

$$\mathbb{P}^N\left\{S \in \Delta^N : \mathbb{P}\left\{\delta \in \Delta : x_N^* \notin \bigcap_{i=1}^m X_i(\delta)\right\} \leq \widetilde{\varepsilon}\right\} \geq 1 - \beta. \qquad (16)$$

- ... but
  - will not show **any** equation
  - will give a **systems' perspective** to smart cities

What makes a city smart?

- (my) Definition: A city is **smart** if “it”
  - **exploits** technology to **advance** operations and services
  - Goals: safety, efficiency and sustainability

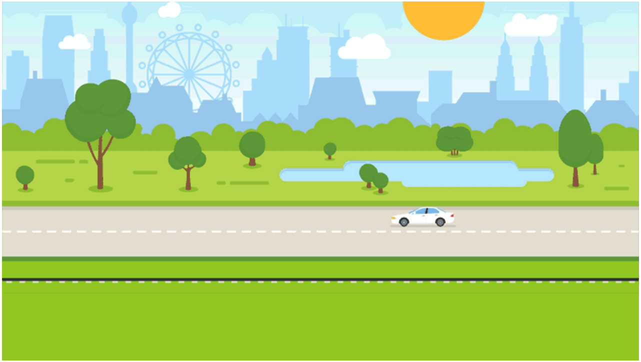


Figure taken from `https://newsroom.cisco.com/`

What makes a city smart?

- To achieve the goals of the future smart cities combine
  - Information and communication technology (ICT)
  - Data collection from citizens, devices, buildings ... and processing
  - Connectedness: vehicles + services + users

Figure taken from https://www.nokia.com/

Smart city: key urban market verticals

UK's industry in the smart cities arena

1. Transportation management
2. Energy management
3. Water management
4. Waste management
5. Assisted living

- Estimated global market of >$400 Billion in 2020[1]
- Catapult Connected Places: UK's innovation accelerator for cities, transport & place leadership[2]

[1] Report Ove Arup & Partners Ltd, Dept for Business Innovation & Skills

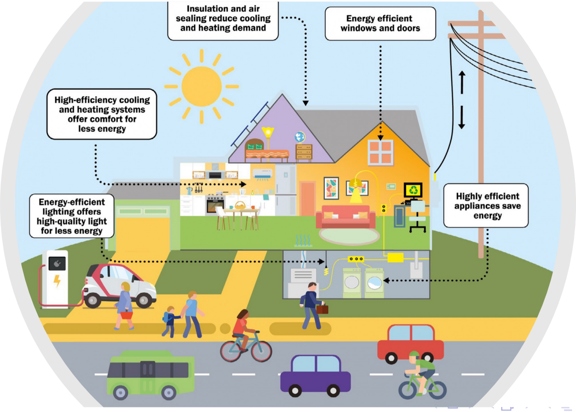
[2] https://cp.catapult.org.uk/

Transportation management

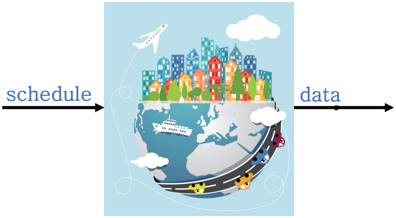
- Connected transportation: informed user choices of how and when they access transport, reduced congestion, ...
- Shift to sustainable transportation: limits carbon emissions and waste, uses renewable resources
- Shared mobility systems: reduce urban density

Energy management: Electricity, heating & cooling

- Building energy management: monitoring and control of heating, ventilation & air conditioning, lighting ...
- Consumption savings through smart meters and efficient appliances
- Consumers become prosumers

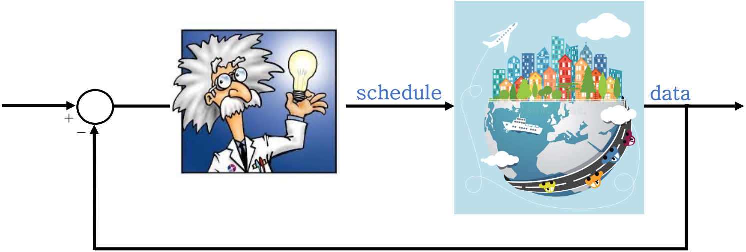


A systems' perspective



- Smart city: plant/process
- Data: output, sensors' measurements
- Schedule: input, actuation

A systems' perspective

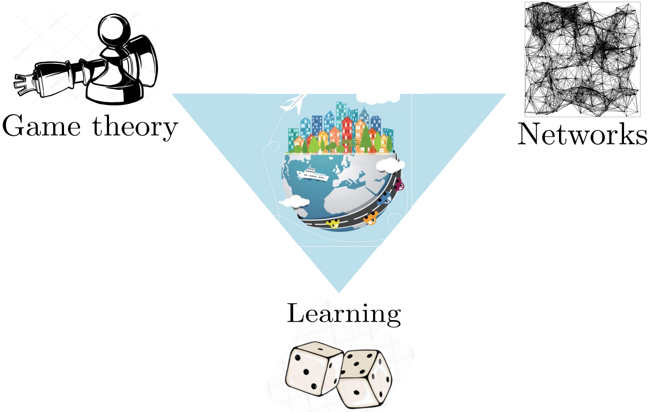


- Smart city: plant/process
- Data: output, sensors' measurements
- Schedule: input, actuation
- How to achieve schedule from seeing data? **Feedback!**

... but there are major challenges!

Math tools

- Networks: Increased levels of connectedness
- Game theory: Strategic behaviour and selfishness
- Learning: Randomness due to uncertainty but availability of data



Hybrid electric vehicle scheduling game

Mani players: Hybrid electric vehicles

- Find optimal schedule but rational
- Price responsive
- Keep local preferences/ limits private

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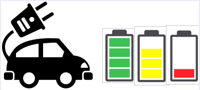

- Find optimal schedule but rational
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Main players: Aggregator

- Aggregates (sums) total demand
- Sets price
- Broadcasts price to vehicles

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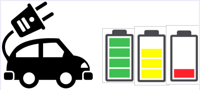

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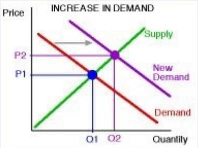
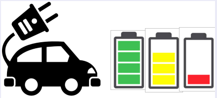
Game rules: Price-demand curve



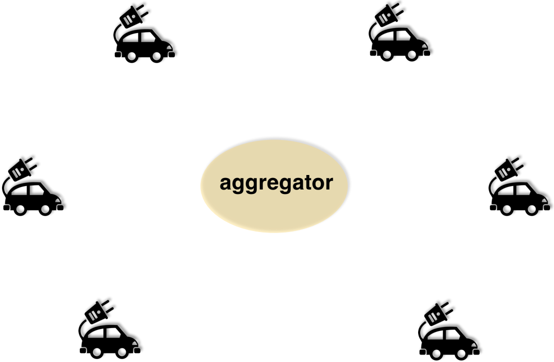
- Increase in demand leads to a higher price
- Elastic demand

Equilibrium seeking algorithm

Step 1: Local computation



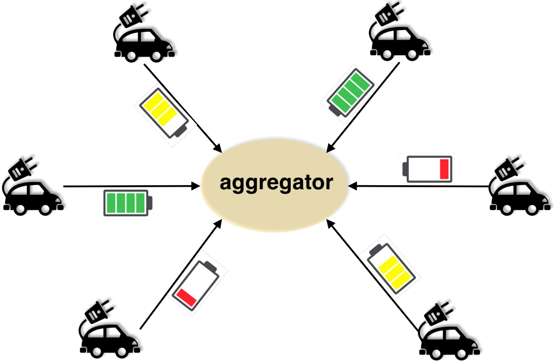
Each vehicle computes in a best-response fashion a tentative charging schedule



Equilibrium seeking algorithm

Step 2: Communication – from vehicles to aggregator

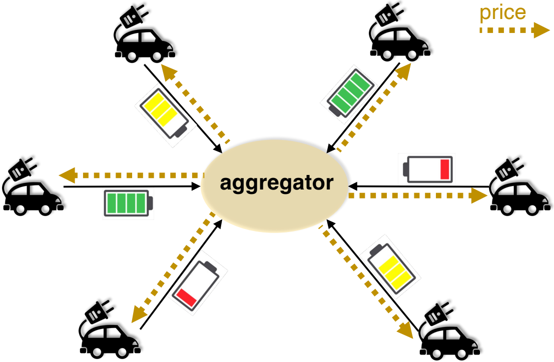
Electric vehicles broadcast their charging schedules to aggregator



Equilibrium seeking algorithm

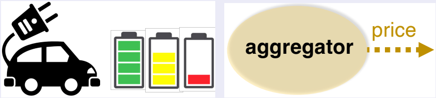
Step 3: Communication – from aggregator to vehicles

The aggregator broadcasts price according to total demand-price curve

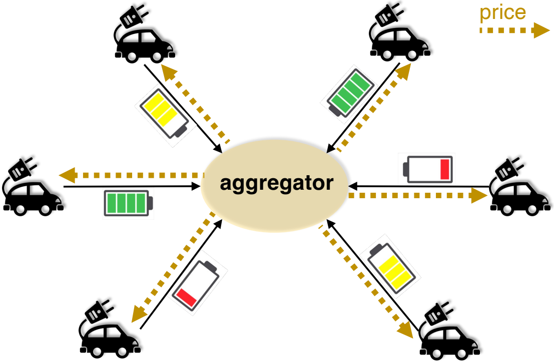


Equilibrium seeking algorithm

Step 4: GO TO Step 1 and REPEAT

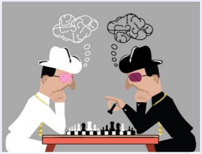


Vehicles compute charging schedules on the basis of price received

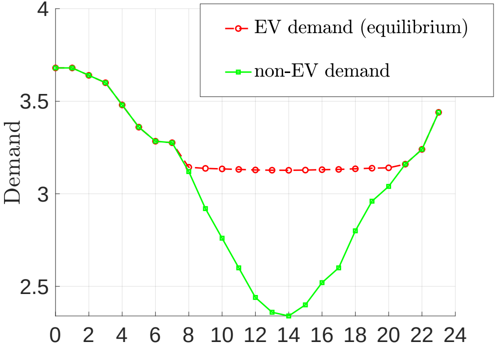


Do we reach an equilibrium? What does this mean?

Main result – Such an iteration:

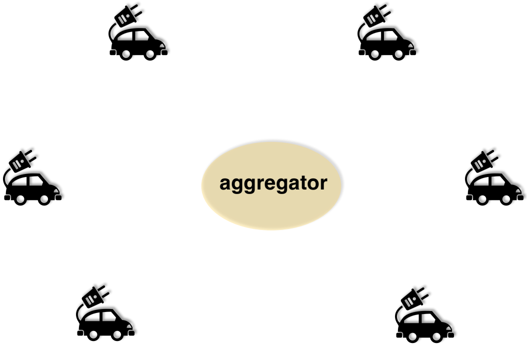


- Converges to an equilibrium charging schedule; no vehicle has incentive to deviate
- Respects privacy requirements
- Is “valley-filling”



A fully distributed set-up

- No aggregator!
- Communication only with neighboring vehicles
- Maintain a local price estimate at the vehicle level



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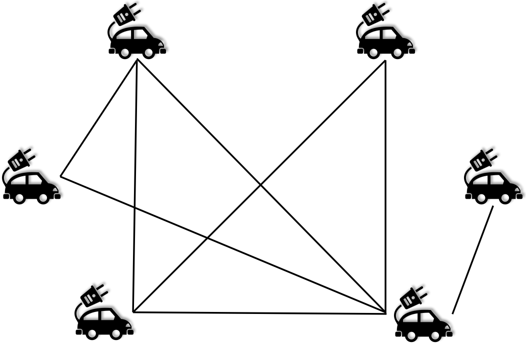

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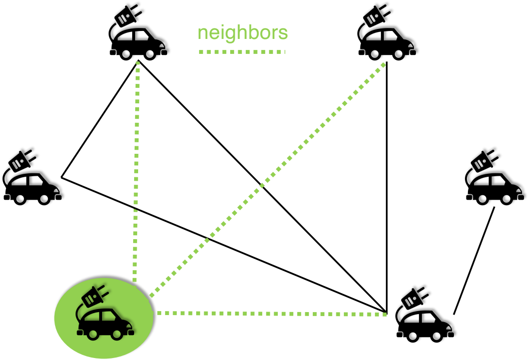

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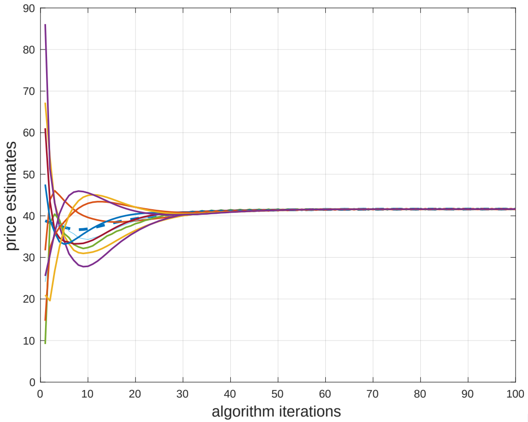
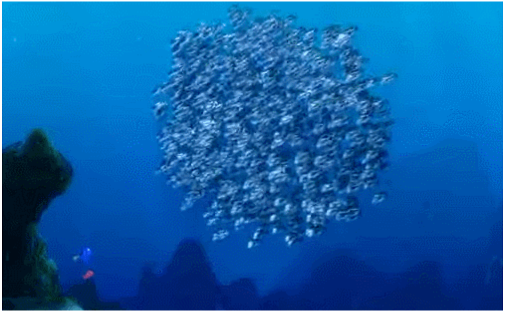
Price estimates reach consensus!



Equilibrium efficiency or else ... price of anarchy

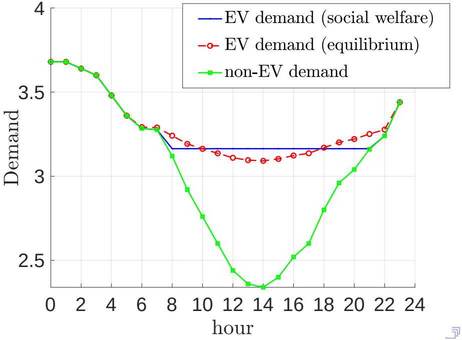
- Motivation from fish or birds
  - Many individuals acting selfishly but the population could do something meaningful – the social welfare!
  - Price of anarchy: "distance" between individuality and social welfare
  - Price of anarchy in the limit, i.e. in large populations?



Equilibrium efficiency or else ... price of anarchy

- Social welfare: Optimum for population if all vehicles cooperate
- Equilibrium: No incentives for vehicles to change their schedule
- As number of vehicles increases, price of anarchy tends to zero!

$$\textit{equilibrium} \xrightarrow{\#\text{vehicles} \uparrow} \textit{social welfare}$$



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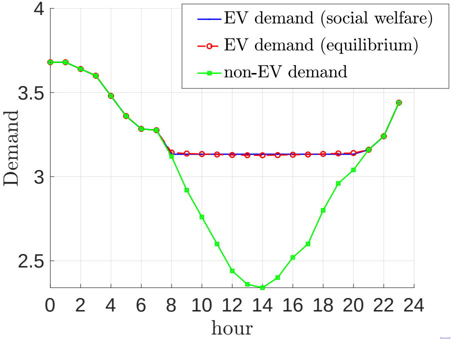
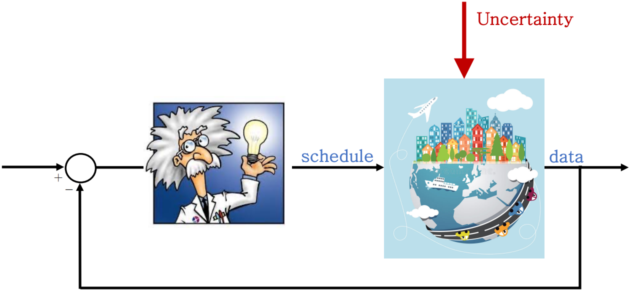


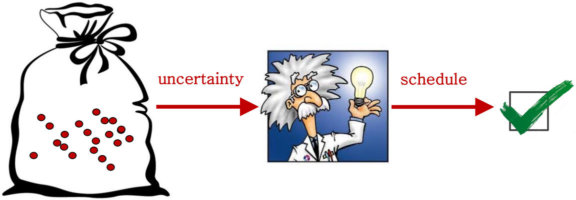
Uncertain environment and data

- Smart cities affected by endogenous and/or exogenous uncertainty



Uncertain environment and data

- Smart cities affected by endogenous and/or exogenous uncertainty
- ... but we have data!



- Schedule depends on data $\Longrightarrow$ random!
- Learning decisions from data!

Uncertain environment and data

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- Schedule depends on data $\implies$ random!
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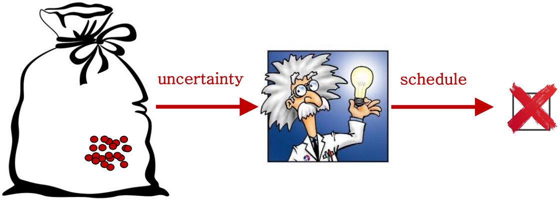
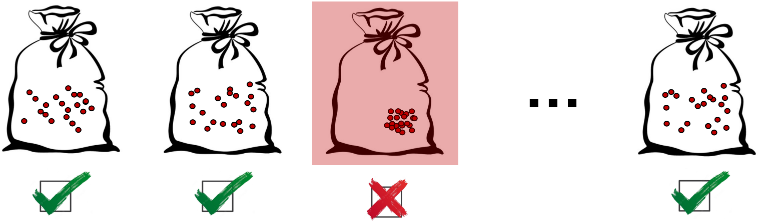


- Schedule depends on data $\Longrightarrow$ random!
- Learning decisions from data!

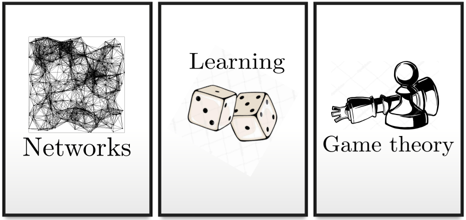
Learning with guarantees

- What does a good schedule mean?
  - How well it performs when it comes to **new** data
- How likely is it to make good schedules for all data-bags?
  - Not possible, but we can guarantee this for most of the data-bags, i.e. in probability
  - *A priori* quantified confidence on the learned schedule!

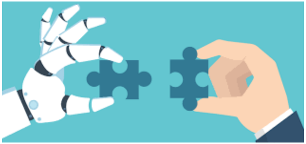


Summary

- Moving to a smart-city paradigm exhibits several challenges that call for math tools



- Other key factors: Socio-political issues; poverty levels; ethical issues & interaction with humans



The book of nature is written in the language of mathematics.

– Galileo Galilei, 1564 – 1642

Thank you for your attention!

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